MATH 103B – Discussion Worksheet 1
March 6, 2023

Attendance policy: Attendance in discussion sessions for this course is *optional*. You are welcome to attend any of the following sections:

1. B01: 12:00-12:50 pm WLH 2209
2. B02: 1:00-1:50 pm WLH 2209
3. B03: 2:00-2:50 pm WLH 2209
4. B04: 5:00-5:50 pm CENTR 207.

You will be awarded 0.2% for each discussion you attend (up to 1% total of extra credit will be awarded). Please make sure your attendance is recorded properly during each session.

Topic: Group actions (Judson 14.1)

Demo Examples

Recall the definition of a **(left) action** of a group G on a set X. We have the following two examples.

Example 0.1. Let $G = \mathrm{GL}_2(\mathbb{R})$ be the group of 2×2 invertible matrices with $\mathbb{R}$-coefficients, and $X = \mathbb{R}^2$. Then G acts on X by left multiplication. More generally, we have $\mathrm{GL}_n(\mathbb{R})$ acts on $\mathbb{R}^n$ by left multiplication.

Example 0.2 (Left translation)**.** Let G be any group and $X = G$. Then G acts on itself by left multiplication, i.e. $(g, x) \mapsto \lambda_g(x) = gx$, where λ_g is left multiplication. Consider the right multiplication $\widetilde{\lambda}_g(x) = xg$. Notice that $\widetilde{\lambda}_g$ is *not* a left action of G on itself.

Discussion Problems

Work in a group of 3-4 students on the following problems.
and Recall the conjugation action of H on G (c.f. Judson Example 14.4).

Problem 1. Let G be a group. Recall the following definitions:

1. Let H be a subgroup of G. The **conjugation action** of H on G (c.f. Judson Example 14.4).
2. Let X be any G-set. For $g \in G$, the **fixed point set** of g in X, denoted by X_g.
3. For any $x \in X$, the **stabilizer subgroup** of x, denoted by G_x.

Problem 2. If G is an abelian group, what is the conjugation action? What is the fixed set of any $g \in G$? What about the stabilizer group of every $x \in G$?

Problem 3. For G be any group acting on itself by left translation, what is the fixed set of $g \in G$? What about the stabilizer group of g?

Problem 4. Let $G = Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$, the quaternion group, and G acts on itself by conjugation. What is the fixed point set of i What is the stabilizer group of $-1, j$, respectively?